Supersonic Annular Nozzles

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An analytical method is presented for the design of axisymmetric annular nozzles to yield uniform and axial flow. Computed results are used to display typical matched inner and outer contours, the effects of major initial flow variables, and the resulting wall forces. For axial inflow, the inner contour is longer than the matched outer contour, and there is an apparent maximum expansion area ratio. Variations in the initial flow angle can be used to vary the base height required from matching considerations, to effect the relative division of thrust and length between inner and outer contour. Previous plug-type nozzles are simple limiting cases of the annular nozzles described here.

Introduction

VARIOUS types of plug and inverted plug nozzles (Figs. 1a,1b,1c) have been studied for air breathing and rocket propulsion,[1-5] and are used in wind-tunnel design[6] (Fig. 1d). In all of these cases, isentropic contours can be obtained for producing uniform and axial flow by using the method of characteristics. The Mach lines shown in Fig. 1 indicate schematically the procedure used and the resulting geometry for each of the designs. In essence, the contours are designed to accept the expansion waves emanating from point P.

A review of the designs shown in Fig. 1 indicates several geometric limitations imposed by gas dynamic considerations. In this and subsequent discussions, axisymmetric and isentropic flow is assumed. For the external expansion plug nozzle (Fig. 1a), expansion waves centered at point P redirect the initial flow to an axial direction. Based on supersonic flow theory[7] the selection of Mach numbers (initial M_1, final M_e) and point P determine the initial flow area (A_i) and flow inclination. For sonic conditions, large angles are required, and an upstream projected area larger than A_e must be provided. With the internal-external expansion plug[5] (Fig. 1c), more freedom is allowed in the choice of inlet flow direction, but again, the exhaust area is limited since the flow must be inclined toward the axis prior to the final expansion (point P outer cowl). The inverted plug (Fig. 1b), contains an internal corner (point P) which is used to expand the flow and direct it axially.

There is a restriction, however, on the minimum radial location of point P (to provide sufficient flow area upstream). Also, with an inverted plug, small inlet areas lead to large base areas (relative to A_e) that create large regions of undefined flow. The internal expansion plug (Fig. 1d) is identical to conventional bell nozzle design[8] for planar flow. For axisymmetric flow, the location of point P and the inlet conditions (A_i and M_i) provide a maximum amount of expansion (zero-base height) with the upper contour serving merely as a reflection surface for Mach waves emanating from point P.

Because of the preceding restrictions with previously available annular nozzles, it may be desirable to investigate a design such as shown in Fig. 2. That is, with given inlet conditions (Y_L, Y_U, $M_i\theta_i$), expansion surfaces are required for both inner and outer contour to produce a uniform and axial exhaust flow. For the nozzle shown in Fig. 2, no methodology has appeared in the available literature. Such a nozzle would produce the minimum area ratio for a given thurst, and truncation or variational[9] techniques could also be applied for minimum length or surface area considerations.

This paper illustrates the principal features of an analytical method for the design of annular supersonic nozzles.

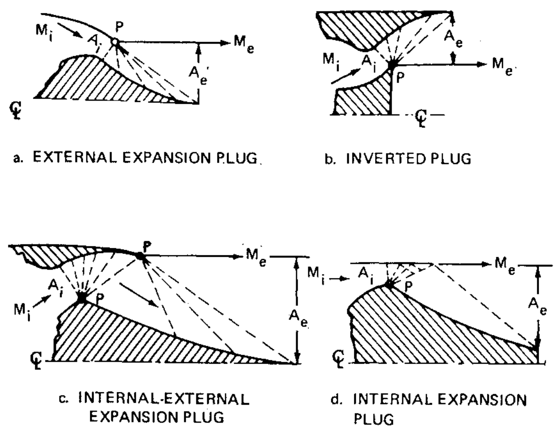

Fig. 1 Simple annular nozzle.

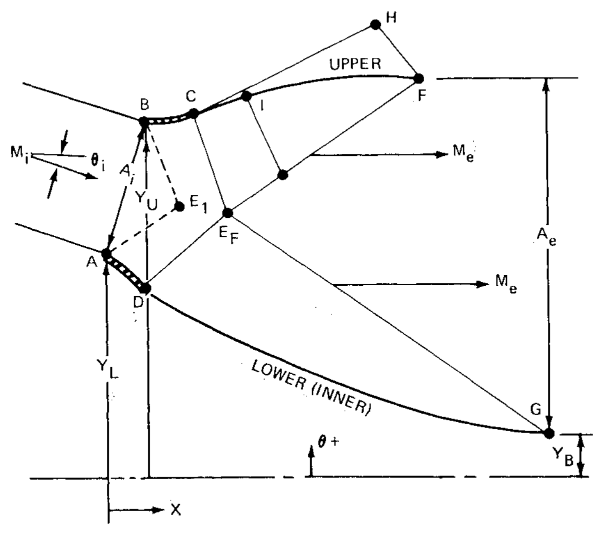

Fig. 2 Nomenclature for annular nozzle.

Received January 18, 1971; presented as Paper 71-43 at the AIAA 9th Aerospace Science Meeting, New York, January 25–27, 1971; revision received September 10, 1971. This work was partially sponsored by an Applied Physics Laboratory (Johns Hopkins University) contract to Propulsion Sciences Inc., Melville, New York.

Index categories: Nozzle and Channel Flow; Hypersonic Air-breathing Propulsion.

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Calculation Procedure

A numerical integration of the governing characteristic equations is used to calculate the complete nozzle flowfield. The general form and method of solution is well-known; for completeness, the equations actually programed are listed.

Along Mach Lines (±), and Streamlines

$$(dy/dx) \pm = (t \pm A)/(1 \mp tA)$$

$$a(\ln p) \pm = \gamma(1 + A^2)dt(/A(1 + t^2) - \varepsilon t\gamma(1 + A^2)dx/y(1 \mp tA)$$

$$T^{\gamma/(\gamma-1)}/p = \text{const.}$$

$$V^2 + 2R\gamma T/(\gamma - 1) = \text{const.}$$

Where t, A, γ, p, ε, T, V, and R are respectively, tangent of the flow angle θ, tangent of the Mach angle, local isentropic coefficient (function of temperature only), pressure, 0 or 1 for planar or axisymmetric flow, temperature, velocity, and specific gas constant (assumed constant throughout). The sound speed is calculated as $(\gamma RT)^{1/2}$, and M denotes the Mach number.

An outline of the calculation procedure follows. Reference should be made to standard texts[10] for a review of the application of the method of characteristics to various problems in supersonic flow.

Assume that uniform and axial flow at a specified value of M_e (or area ratio) is desired along the noncharacteristic line A-B (Fig. 2). The initial radial position (Y) of the nozzle at the upper (Y_u) and lower wall (Y_L) are arbitrary as well as M, T, and P along A-B. The only restriction imposed on the initial data is that it does not lead to a shock in the zone of influence A-E_1-B. Downstream of the initial line, expansions are required along both the upper and lower contours. The amount of expansion, however, is unknown (i.e., points C and D are unknown), and the expanding surfaces (A-D and B-C) are arbitrary and may be sharp corners. (This is analogous to previous procedures[8] for bell nozzles.) Points C and D are to be determined so that the intersection of Mach waves D-E_F and C-E_F produces axial flow ($\theta = 0$) at point E_F and the desired exit Mach number (M_e). Along Mach lines $E_F - F$ and $E_F - G$ we impose the condition $\theta = 0$, and the terminal points F and G are to be streamlines emanating from C and D, respectively. A typical contour point I is located by mass flow integration so that the mass flow through $E_F - C$ and $E_F - I$ are the same. This procedure requires considerable attention in determining the accurate location of point E_F; otherwise, no major calculation difficulties are encountered.

Results

Typical Contours

A set of typical matched contours is shown in Fig. 3, along with the location of point E_F (Fig. 2). An initial sharp corner for both upper and lower contours is assumed. It is quite evident that with increasing area ratios the lower contours are longer than the corresponding upper contours. This results from mass continuity requirements along the last Mach lines in axisymmetric flow. When both upper and lower contours are far from the axis (or in planar flow, $\varepsilon = 0$), the mass flow through $E_F - F$ and $E_F - G$ is the same and points F and G are equidistant from E_F.

Axisymmetric Effects

Three other important axisymmetric results only partially evident from Fig. 3 are: larger initial expansion angles are required for the lower contour than the upper contour, the $\theta = 0$ streamline (points E_F) curves toward the axis with increasing expansion area ratios, and there is an apparent maximum area ratio for each set of initial conditions. The first

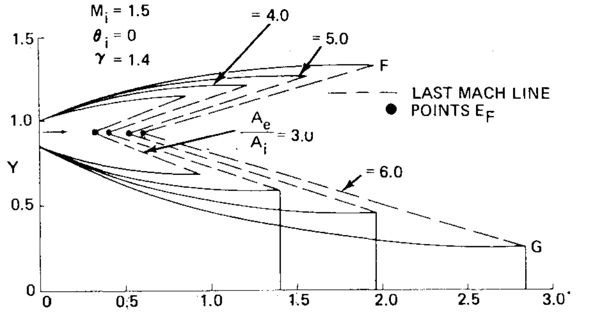

Fig. 3 Sample contours.

two results are due to streamlines converging near the upper wall and the additional turning required along the lower wall to compensate for this effect. The last result is partially independent of axisymmetric effects in the initial expansion region (D-E_F-C in Fig. 2), in that even if the $\theta = 0$ streamline were in the middle of the initial annulus there would be a maximum flow area available between point E_F and the axis. This geometric limitation is shown for a number of computed matched contours in Fig. 4. In this figure values along the abscissa correspond to various assumed inlet annular locations, and the ordinate is the required base height for the desired area ratios (Ae/Ai). As the lower wall approaches the axis ($Y_L = 0$), there is a very limited range of annular nozzles possible (Fig. 4); for each value of Y_L/Y_u there is only one area ratio that will produce a zero base height ($Y_B = 0$)—this is the apparent maximum area ratio possible within the restrictions of uniform and isentropic flow assumed.

Effects of Initial Flow Angle

In addition to annulus height, the initial flow inclination angle is one of the most important parameters in annular nozzle design. As shown in Fig. 5, the major differences between external expansion plug, inverted plug, and annular nozzles can be correlated with this parameter. For the external expansion plug with fixed annulus location (Y_L/Y_u), there is a unique maximum throat inclination. As the throat inclination is reduced, there is a limited and unique relationship between the base height and area ratio (double dashed lines in the lower right hand corner of Fig. 5). The other extreme of single expansion nozzles is the inverted nozzle shown in the upper left hand corner of Fig. 5. In this case, for each throat inclination (away from the axis) there is a unique area ratio. For all other cases, an annular nozzle is required; that is, the inverted and external expansion plug are actually limiting cases of a general annular nozzle in which the area ratio and

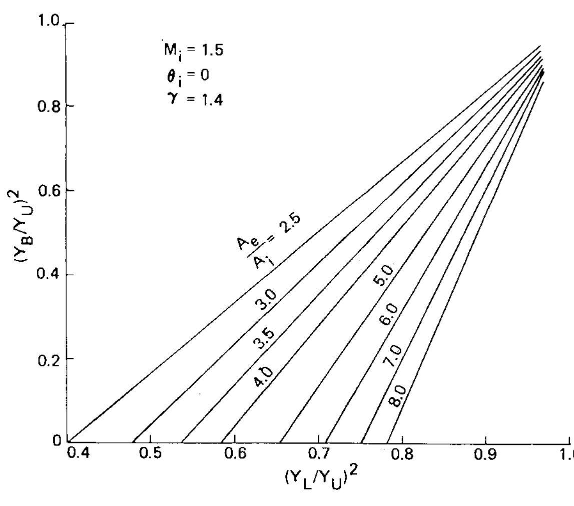

Fig. 4 Geometric restrictions.

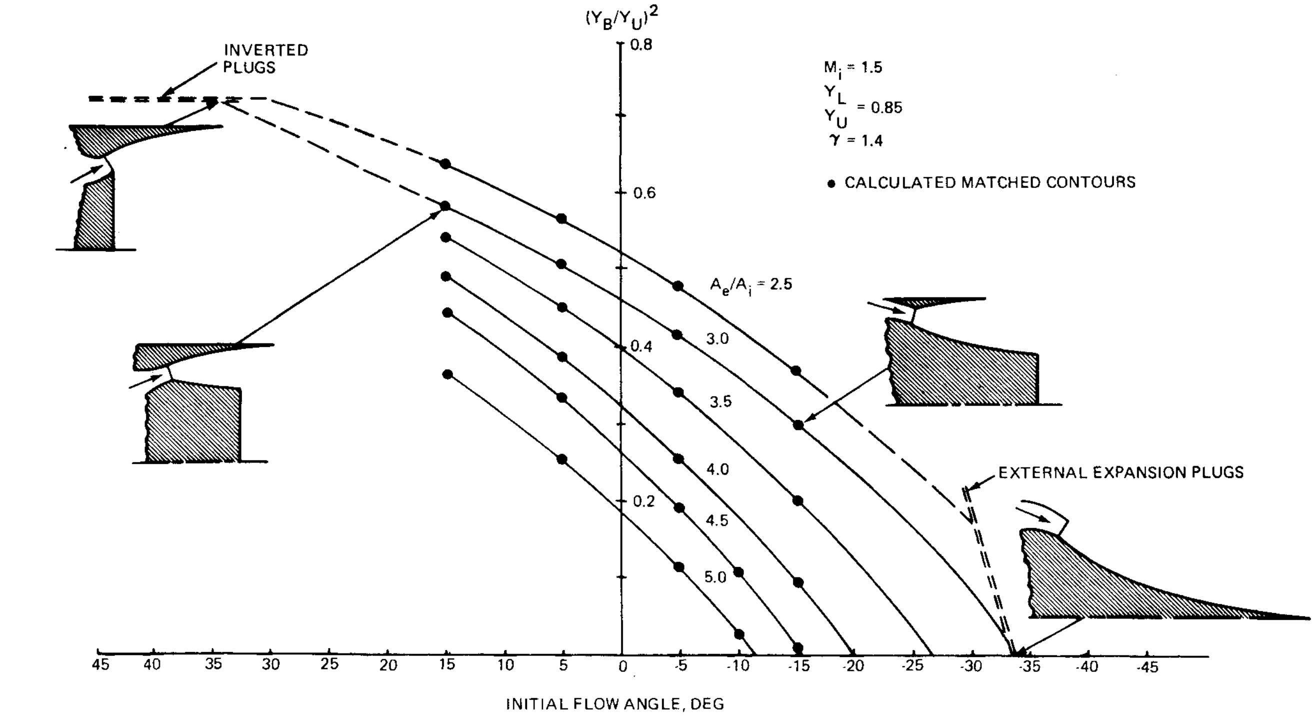

Fig. 5 Effect of initial flow angle.

throat inclination can be independently specified. By comparing Figs. 4 and 5 it will be noted that if with a given annulus height, larger area ratios are desired with a zero-base height the throat should be inclined away from the axis. To reduce the base height with a given area ratio, throat inclinations toward the axis are required.

Effect of M and γ

As shown in Fig. 6 increasing the inlet Mach number, with the same area ratio required, results in smaller initial expansion angles, shallower Mach lines, hence longer nozzles; and a shift in both contours toward the axis. One other variable which affects the annular nozzle contour is γ. The effects previously noted with bell nozzles[8] are qualitatively the same with annular nozzles; that is, decreasing values of γ result in slightly shorter nozzles.

Wall Pressure Forces

Axial thrust coefficients obtained by integrating along the contour added to the initial axial force should, for axial exit flow, be equal to the one-dimensional values directly calculable from the exit Mach number. With this consideration, larger wall pressure forces must accompany large initial flow inclination. The trends shown in Fig. 7 are typical in that positive

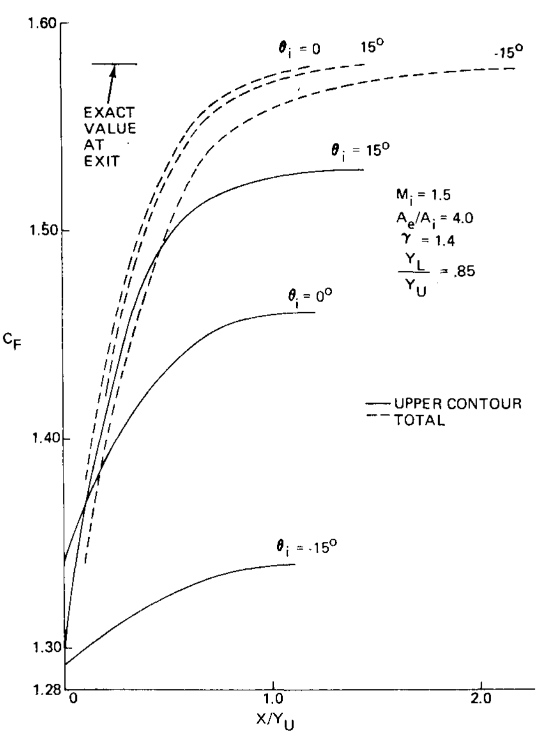

Fig. 6 Effect of initial Mach number.

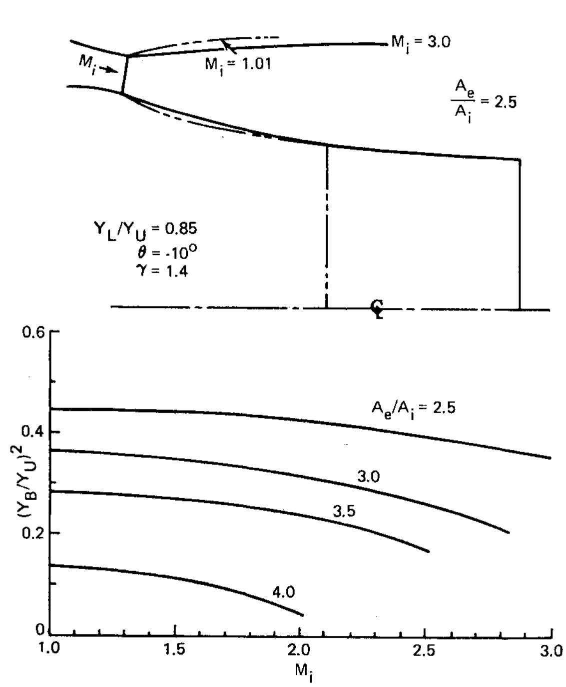

Fig. 7 Axial wall forces.

θ_i leads to larger wall forces along the upper contour while negative θ_i yields large forces along the lower contour.

Discussion

Alternate Method

In addition to the method described previously, one other method has been proposed for the analytical design of contour nozzles. With this alternate method a velocity distribution is given along a reference streamline from which other streamlines, hence nozzle shapes, can be determined. In bell nozzles the axis of symmetry was a natural choice for the reference streamline. Assuming one can prescribe a velocity distribution, which does not lead to undefined regions or unrealistic shapes, the question of where to prescribe the distribution for annular nozzles is difficult to answer from the current study. For $\theta_i = 0$ and small annular heights it appears that a mean line for prescribing the velocity (and $\theta = 0$) through the annulus may yield reasonable contours (Fig. 3). For the case where θ_i is not zero, a curved surface must be chosen, at least for axial flow exhaust nozzles, and the results such as shown in Fig. 5 may aid in determining the minimum length and approximate axial location for such curved surfaces.

Contour Truncation and Extensions

On examining a number of matched contours it was noted that large portions of either nozzle surface may be altered without affecting the pressure distribution of the opposing surface. For example, in Fig. 3 the upper contour may be completely eliminated downstream of the initial sharp corner with no design modification required for the lower contour (a low ambient pressure is assumed). Thus, with one notable exception, the procedure of truncating portions of uniform flow nozzles may be used here to optimize for minimum length or surface area. The exception, of course, refers to the inner contour for which truncation does not led to a complete thrust loss and where semiemperical methods are required to include the base pressure effect.

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